A NEW MATHEMATIC MODEL OF THE DYNAMIC BEHAVIOUR OF THE MECHANICAL SYSTEM OF BALLOON SPINNING

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1. INTRODUCTION

The mechanical system of balloon spinning analyzed in the present study, is formed of the following elements – as represented in the principle diagram plotted in Figure 1:

(1) the reinforced spindle-package assembly, rotating around the vertical axis (Δ) represented only by circle (Γ_w) of normal cross section of the winding package or of bobbin (Bb) which contains the winding point P, and by the spindle's rod (Sp), solidary with circle (Γ_w);

(2) ring (Rn) solidary with the ringrail segment (Rr) corresponding to a spindle - an element which executes an alternative rectilinear translation along axis (Δ) - represented by circle (Γ_T) on which the traveler is moving;

(3) traveller (Tr), executing an ellicoidal motion, composed of a circular movement on circle (Γ_T) an alternative rectilinear translation , executed simultaneously with circle (Γ_T), respectively with the ringrail (Rr).

The present study analyzes only the spinning sub-system formed of traveller-ring – bobbin, point A (of yarn's out put from the feeding rollers) and B (the center of yarn's eye, Ge) representing external connection points imposed to the yarn portion between the feeding roller and the traveller. According to the known methodology of mechanics, the effect of yarn's ABC portion action $(C = Tr)$ on the above mentioned mechanical system is substituted by tension $\bar{T}$ in point C - the value of which is not known, although its orientation is stated by angle ψ^* which it forms with the vertical; this angle may be measured either by direct measurements or it may be calculated.

The mechanics problem involves settlement - by means of Lagrangean formalism - of the motion equations of the mechanical system considered.

2. ELEMENTS OF THE MECHANICAL SYSTEM'S KINEMATICS

Some kinematic aspects will be first discussed. To each of elements (1) and (2), a straight triorthogonal Cartesian mark is invariably attached (Fig.1). Mark $(R'_1) = O'_1 x'_1 y'_1 z'_1$ has its origin in the center of the circle of bobbin (B_b)' s inferior basis, with axis $O'_1 z'_1$ oriented according to axis (Δ); mark $(R'_2) = O'_2 x'_2 y'_2 z'_2$ has its origin in the center of circle (Γ_T), with the $O'_2 z'_2$ axis oriented according to axis (Δ), while axis $O'_2 y'_2$ coincides with the ringrail's longitudinal axis.

Element (3), the traveller – assimilated to a material point has its position determined by its cylindrical coordinates in mark $(R'_c) \equiv C\rho\varphi\varsigma$. The positions of marks (R'_1), (R'_2) and (R'_c) are considered versus the straight, steady, triothogonal Cartesian mark $(R) \equiv Oxyz$, the axes' vectors being $\bar{i}, \bar{j}, \bar{k}$; $O \equiv O'_1$, $Oz \equiv O'_1 z'_1$ ($\bar{k}'_1 \equiv \bar{k}$), $Ox \,//\, O'_2 x'_2$, ($\bar{i}'_2 \equiv \bar{i}$), $Oy \,//\, O'_2 y'_2$, ($\bar{j}'_2 = \bar{j}$).)

Table I lists the position (p.p) and the kinematic (p.c.I, p.c.II) parameters characterizing the motion of the three elements of the mechanical system. T_{Rr} represents the period of ringrail's rectilinear alternative motion, equal to the deposition time of a layer in the bobbin, while φ_c represents traveller's polar angle in mark (R'_c).

Table I, column 2, shows that the total number of time variable position parameters of the mechanical system under analysis is $p_v = \sum_{i=1}^{3} p_{v(i)} = 5$. The occurrence of the following equations of geometric connection between the 5 parameters:

Table 1

$$f_1(x_T, y_T) = x_T^2 + y_T^2 - r^2 = 0;$$

(i)	$p_{v(i)}$	p.p.	p.c.I	p.c. II
1	2	3	4	5
(1)	1	$\varphi_1 = \varphi_1(t)$	$\overline{\omega}_1 = \dot{\varphi}_1 \overline{k}_1' = \dot{\varphi}_1 \overline{k} = \omega_1 \overline{k}$	$\overline{\varepsilon}_1 = \dot{\overline{\varpi}}_1 = \ddot{\varphi}_1 \overline{k}$
(2)	1	$z_2 = z_2(t) = z_2(t + T_{Rr})$	$\overline{v}_{O_2'} = \dot{z}_2 \overline{k}_2' = \dot{z}_2 \overline{k} = v_2 \overline{k}$	$\overline{a}_{O_2'} = \dot{\overline{v}}_{O'_2} = \ddot{z}_2 \overline{k}$
(3)	3	$x_T(t); y_T(t); z_T(t);$ $x_T = r\cos\varphi_c;$ $y_T = r\sin\varphi_c;$ $z_T = z_2(t);$	$\overline{v}_3 = \overline{v}_T = \dot{\overline{r}}_T$ $\overline{v}_3 = \dot{x}_T \overline{i} + \dot{y}_T \overline{j} + \dot{z}_T \overline{k}$ $\overline{v}_3 = r\dot{\varphi}_c(-\sin\varphi_c \overline{i} + \cos\varphi_c \overline{j}) + \dot{z}_2 \overline{k}$	$\overline{a}_3 = \overline{a}_T = \ddot{\overline{r}}_T$

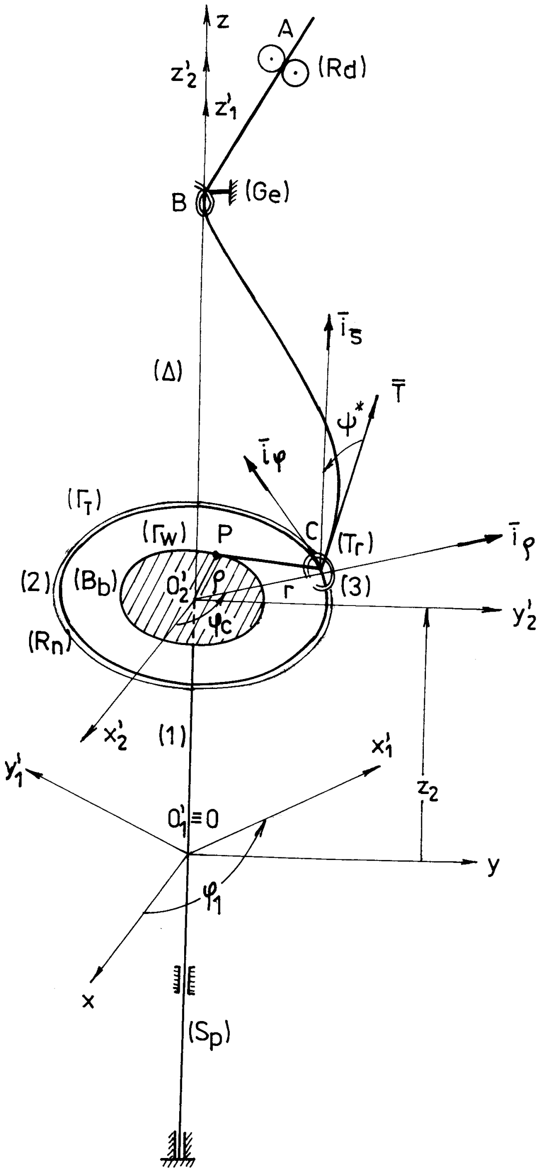

Figure 1. Principle diagram of the balloon spinning mechanical system

$$f_2(z_T) = z_T - z_2(t) = 0$$

induces a reduction in the system/s degrees of freedom to:

$$p = p_v - l_{int} = 3 \qquad (2.1)$$

Consequently, the mechanical system's position is determined by 3 Lagrange coordinates, represented either by 3 of the 5 variable in time parameters or by other, independent ones. Further on, the following 3 generalized coordinates will be considered:

$$q_1 = \varphi_1(t);\ q_2 = z_2(t);\ q_3 = \varphi_c(t) \qquad (2.2)$$

with the corresponding generalized velocities:

$$\dot{q}_1 = \dot{\varphi}_1(t) = \omega_1;$$
$$\dot{q}_2 = \dot{z}_2(t) = v_2;\ \dot{q}_3 = \dot{\varphi}_c(t) = \omega_c. \qquad (2.3)$$

3. DYNAMIC OF THE MECHANICAL SYSTEM

3.1. Calculation of the mechanical system's kinetic energy

Starting from the motions performed by the mechanical system's elements, i.e.: (1), a rotation motion around a steady axis, (2) a rectilinear alternative translation along axis (Δ) and (3) an ellicoidal motion, the expressions of their energies will take the form:

$$E_{c_1} = \frac{1}{2} J_1 \overline{\omega}_1^2 = \frac{1}{2} J_1 \dot{\varphi}_1^2;$$

$$E_{c_2} = \frac{1}{2}M_2\bar{v}^2_{O'_2} = \frac{1}{2}M_2\dot{z}_2^2 = \frac{1}{2}M_2 v_2^2 \qquad (3.1)$$

$$E_{c_3} = \frac{1}{2}M_3\bar{v}_3^2 = \frac{1}{2}M_3 v_T^2 = \frac{1}{2}M_3\left(r^2\dot{\varphi}_c^2 + \dot{z}_2^2\right)$$

where J_1 – the inertia moment of element (1) versus axis (Δ); M_2 and M_3 - the weight of element 2 and 3, respectively.

The kinetic energy of the mechanical system considered, defined by relation $E_c = \sum_{i=1}^{3} E_{c_i}$, takes – as based on relations 3.1 – the final expression:

$$E_c = \frac{1}{2}\left[J_1\dot{\varphi}_1^2 + \left(M_2 + M_3\right)\dot{z}_2^2 + M_3 r^2\dot{\varphi}_c^2\right] \quad (3.2)$$

3.2. Calculation of the mechanical system's generalized forces

Calculation of the generalized forces considered only the external active stresses and the external dissipative passive ones, as plotted in Fig.2 (their values being determined previously, by means of other investigation methods).

The external, active stresses which are manifested act on element (1) are:

- element's weight:

$$\bar{P}_1 = M_1\bar{g} = -M_1 g\bar{k}\ ; \qquad (3.3)$$

- moment of the action engine torque of the mechanical system, transmitted to the spindle by a transmission belt;

$$\bar{M}_{m_1} = \bar{M}_m = M_m\bar{k}\,. \qquad (3.4)$$

The external passive dissipative stresses are repesented by:

- the friction moment during rotation in the pilot bearing (Bp), which takes, in our case, a value of:

$$\bar{M}_{piv} = -\nu M_1 g\bar{k}\,, \qquad (3.5)$$

ν being the coefficient of friction on turning.

- the friction momentum in the cylindrical bearing (Bc) which, under conditions of the material symmetry of element's revolution around the rotation axis, may be considered as constant:

$$\bar{M}_f = -M_f\bar{k}\,, \qquad (3.6)$$

its module, M_f, being possibly approximated with the value of its static moment (um).

The $M_0 = M_{piv} + M_f$ sum remains approximative, with a constant value of the total friction momentum in the two bearings, corresponding to the element's static state, a value that may be determined experimentally.

Consequently, twisting device of the external stresses acting upon element (1) takes the form:

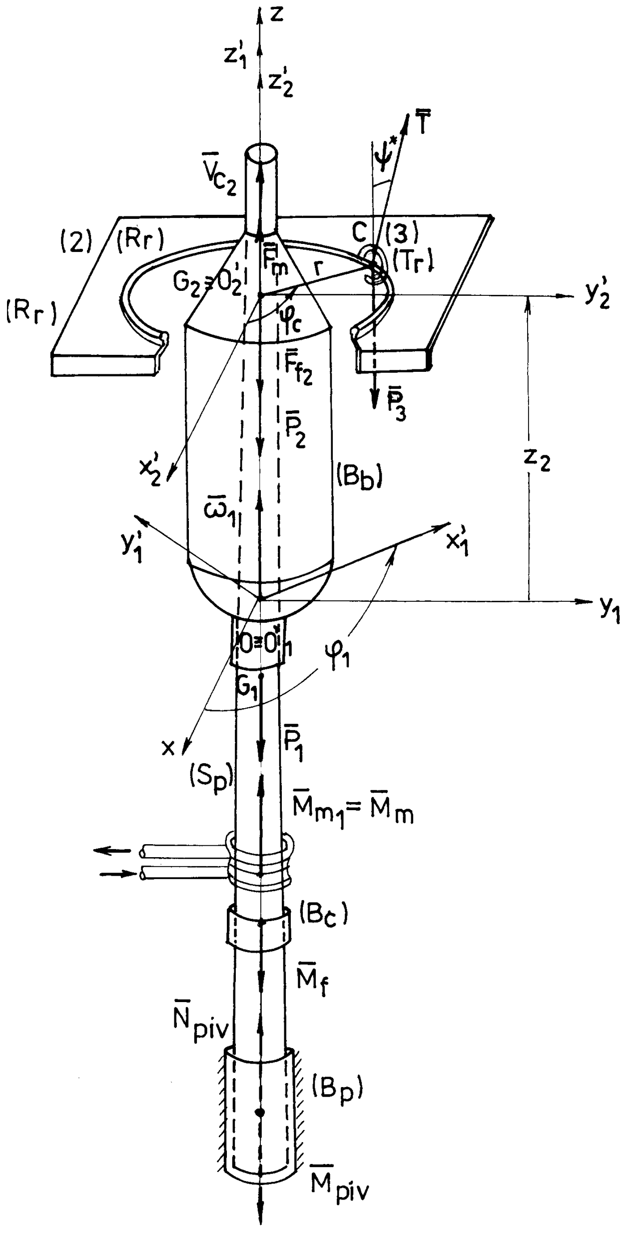

Figure 2. Image of the active external and passive dissipative stress acting on the mechanical system

$$\bar{T}_{G_1}\left(\bar{F}_1\right) = \begin{cases} \bar{F}_1 = \bar{P}_1 = M_1\bar{g} = -M_1 g\bar{k} \\ \bar{M}_1 = \bar{M}_m + \bar{M}_{piv} + \bar{M}_f = \left(M_m - M_0\right)\bar{k} = M_1\bar{k} \end{cases} \qquad (3.7)$$

On considering relations $\bar{V}_{G1} \equiv 0$; $\bar{\omega}_1 = \dot{\varphi}_1\bar{k}$, the following conditions result:

$$\overline{F}_1 \frac{\partial \overline{v}_{G_1}}{\partial \dot{\varphi}_1} = \overline{F}_1 \frac{\partial \overline{v}_{G_1}}{\partial \dot{z}_2} = \overline{F}_1 \frac{\partial \overline{v}_{G_1}}{\partial \dot{\varphi}_C} = 0 ;$$

$$\overline{M}_1 \frac{\partial \overline{\omega}_1}{\partial \dot{\varphi}_1} = M_1 ; \ \overline{M}_1 \frac{\partial \overline{\omega}_1}{\partial \dot{z}_2} = M_1 \frac{\partial \overline{\omega}_1}{\partial \dot{\varphi}_C} = 0 . \quad (3.8)$$

The external active stresses acting upon element (2) are:

- element's weight:

$$\overline{P}_2 = M_2 \overline{g} = -M_2 g \overline{k} ; \quad (3.9)$$

- the driving force putting the assembly into motion:

$$\overline{F}_m = F_m \overline{k} . \quad (3.10)$$

The external passive dissipative forces acting upon element (2) are represented by the friction forces to sliding in the translation couples, that permit railring's movement. The resulting unique force, $\overline{F}_{f_2}$, has its support represented by axis $O'_2 z'_2$, its module F_0, determined experimentally for the static case is considered constant and is permanently oriented counter-clockwisely to the direction of velocity $\overline{v}_{G_2} = \dot{z}_2 \overline{k}$; consequently, this force will be expressed as:

$$\overline{F}_{f_2} = -sign\ \dot{z}_2 \cdot F_0 \cdot \overline{k} = \widetilde{F}_0 \overline{k} \quad (3.11)$$

Thus, the wrench in pole G_2, of all external forces that act upon element (2), will take the form:

$$T_{G_2}\left(\overline{F}_2\right) = \begin{cases} \overline{F}_2 = \overline{P}_2 + \overline{F}_m + \overline{F}_{f_2} = \left(F_m - M_2 g - sign\ \dot{z}_2 \cdot F_0\right)\overline{k} \\ \overline{M}_0 = 0 \end{cases} \quad (3.12)$$

On considering relations $\overline{v}_{G_2} = \dot{z}_2 \overline{k}$; $\overline{\omega}_2 = 0$, the following conditions will result:

$$\overline{F}_2 \frac{\partial \overline{v}_{G_2}}{\partial \dot{\varphi}_1} = 0 ; \ \overline{F}_2 \frac{\partial \overline{v}_{G_2}}{\partial \dot{z}_2} = F_2 ; \ \overline{F}_2 \frac{\partial v_{G_2}}{\partial \dot{\varphi}_C} = 0 . \quad (3.13)$$

Element (3), the traveller, is driven by two external forces:

- weight

$$\overline{P}_3 = M_3 \overline{g} = -M_3 g \overline{k} ; \quad (3.14)$$

- tension

$$\overline{T} = T\left[sin\psi^* \cos\varphi_c \overline{i} + sin\psi^* \sin\varphi_c \overline{j} + \cos\psi^* \overline{k}\right] \quad .15)$$

in the hypothesis of the plane shape of the yarn portion forming the balloon, the position situated in the plane determined by the symmetry axis of ring's revolution and also by the traveller.

The analytical expression in the steady mark of the resultant of the external forces acting upon the traveller take the form:

$$\overline{F}_3 = \overline{P}_3 + \overline{T} = T\left[sin\psi^* \cos\varphi_c \overline{i} + sin\psi^* \sin\varphi_c \overline{j}\right] + \\ + \left(T \cos\psi^* - M_3 g\right) \overline{k} \quad (3.16)$$

On considering relation $\overline{v}_3$ from table I, column 4, the following conditions result:

$$\overline{F}_3 \frac{\partial \overline{v}_3}{\partial \dot{\varphi}_1} = 0 ; \ \overline{F}_3 \frac{\partial \overline{v}_3}{\partial \dot{z}_2} = F_{3z} = T \cos\psi^* - M_3 g ;$$

$$\overline{F}_3 \frac{\overline{v}_3}{\partial \dot{\varphi}_C} = 0 . \quad (3.17)$$

Relations (3.8), (3.13) and (3.17) lead is the following final expressions of the three generalized forces:

$$Q_1 = \sum_{i=1}^{3} \left(\overline{F}_i \frac{\partial \overline{v}_{G_i}}{\partial \dot{\varphi}_1} + \overline{M}_i \frac{\partial \overline{\omega}_i}{\partial \dot{\varphi}_1} \right) = M_1 = M_m - M_0 ;$$

$$Q_2 = \sum_{i=1}^{3} \left(\overline{F}_i \frac{\partial \overline{v}_{G_i}}{\partial \dot{z}_2} + \overline{M}_i \frac{\partial \overline{\omega}_i}{\partial \dot{z}_2} \right) = F_2 + T \cos\psi^* - M_3 g =$$

$$= F_m - \left(M_2 + M_3\right) g + T \cos\psi^* - sign \cdot \dot{z}_2 \cdot F_0 ; \quad (3.18)$$

$$Q_3 = \sum_{i=1}^{3} \left(\overline{F}_i \frac{\partial \overline{v}_{G_i}}{\partial \dot{\varphi}_C} + \overline{M}_i \frac{\partial \overline{\omega}_i}{\partial \dot{\varphi}_C} \right) = 0 .$$

3.3. Lagrange equations of species II, corresponding to the mechanical system

For completing the system of Lagrange equations that describe the dynamic behaviour of the mechanical system:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_k}\right) - \frac{\partial E}{\partial q_k} = Q_k \quad (k=1,2,3) \quad (3.19)$$

the partial derivatives of kinetic energy (3.2) are first calculated, versus the generalized coordinates and generalized velocities:

$$\frac{\partial E_c}{\partial \varphi_1} = \frac{\partial E_c}{\partial z_2} = \frac{\partial E_c}{\partial \varphi_c} = 0 ; \quad (3.20)$$

$$\frac{\partial E_c}{\partial \dot{\varphi}_1} = J_1 \dot{\varphi}_1;\ \frac{\partial E_c}{\partial \dot{z}_2} = (M_2 + M_3)\dot{z}_2;$$

$$\frac{\partial E_c}{\partial \dot{\varphi}_c} = M_3 r^2 \dot{\varphi}_c$$

On considering conditions (3.20), system (3.19) will be reduced to a system of the form:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_k}\right) = Q_k\ (k=1,2,3) \qquad (3.21)$$

On introducing expressions (3.18) and (3.20) in (3.21), the following system of Lagrange equations of species II will be obtained:

$$\ddot{\varphi}_1 = \frac{1}{J_1}(M_m - M_0);\ \ddot{\varphi}_c = 0;$$

$$\ddot{z}_2 = \frac{1}{M_2 + M_3}\left[F_m + T\cos\psi^* - (M_2 + M_3)g - sign \cdot \dot{z}_2 \cdot F_0\right] \qquad (3.22)$$

The previous system of ordinary differential second order equations, the following initial conditions will be associated for functions $\varphi_1(t)$, $z_2(t)$ and $\varphi_c(t)$.

- position conditions:

$$\varphi_1(0) = \varphi_1^0;\ z_2(0) = z_2^0;\ \varphi_c(0) = \varphi_c^0; \qquad (3.23)$$

- velocity conditions

$$\dot{\varphi}_1(0) = \overline{\omega}_1^0;\ \dot{z}_2(0) = v_2^0;\ \dot{\varphi}_c(0) = \omega_c^0. \qquad (3.24)$$

4. CONCLUSIONS

The above presented mathematic model, represented by system (3.22) of differential equations and also by sets of initial and conditions, permits the resolution of two types of problems of the dynamics characterizing the spinning mechanism on the ring spinning machine:

a) The fundamental-type problem, which studies the operation of the spinning system in a transitory regime, such as the one corresponding to machine's starting phase - a case in which the initial conditions of speed take the forms

$$\omega_1^o = 0;\ v_2^o = 0;\ \omega_c^o = 0; \qquad (4.1)$$

which indicate the normal position of the system's elements - or of the one corresponding to machine's stop phase, when the constants expressing the initial speeds take the values

$$\omega_1^o = \omega_1;\ v_2^o = v_2;\ \omega_c^o = \omega_c; \qquad (4.2)$$

corresponding to the operation of the ring-spinning machine in a stationary regime;

b) The direct-type problem, corresponding to machine's operation in a permanent regime, equations (3.22) permitting the establishment of the conditions that should be satisfied by the stresses - the motor one especially - that act on the system, so that the operation of yarn's torsion should occur as a stationary process.

A necessary condition for a normal operation of the mechanical spinning system is that element (1), the spindle-bobbin assembly, should execute an uniform rotation. Once this condition met, there results that:

$$\ddot{\varphi}_1 \equiv 0;\ \dot{\varphi}_1 = \overline{\omega}_0 = const.$$

Under such conditions, equation Lagrange provides the dependence law of the twisting moment on the angular speed and possibly, that of the rotation angle, which might permit identification of the mechanical characteristic of the system's driving engine.

Also, the same equation may provide the data necessary for the automatic regulation of the driving engine's rotative speed.

References

[1] ***De Barr A.E., Catling H.*** *(1965):"Manual of Cotton Spinning" vol. V. "The Principles and Theory of Ring Spinning", Ed. F. Charnley and P. W. Harrison, London.*

[2] ***Buzescu F. L., Cuzic Zvonaru C., Irimiciuc N.*** *(1998): "Modele matematice în tehnică filării cu balon", Ed. Cermi, Iasi.*

[3] ***Buzescu F. L., Fetecau C., Cuzic Zvonaru C., Irimiciuc N.*** *(1999): "Un modello matematico dell' avvolgimento conico dei fili tessili", Revista delle TECNOLOGIE TESSILI no. 3.*

[4] ***Fraser W. B.*** *(1993): "On the Theory of Ring Spinning", Phil Trons. R. Soc. Lond. 342.*

[5] ***Mangeron D., Irimiciuc N.,*** *(1978, 1980): "Mecanica rigidelor cu aplicatii in inginerie", Ed. Tehnica, Bucuresti, Romania, vol.I, vol II.*