CONTRIBUTIONS TO THE SYNTHESIS OF THE WATT AND STEPHENSON MECHANISMS WITH ADJUSTABLE IN STEPS LINKS

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1. INTRODUCTION

The six link mechanisms which have at least an element with a adjustable in steps length, from an topological point of view are classified in Watt and Stephenson mechanism. Depending on the basis rank the Watt mechanism has two alternatives: Watt I when the basis has the rank 2 (binary) and Watt II on the basis has the rank 3 (ternary) [1]. The Stephenson mechanism has three topological alternatives [1], [4]. These are Stephenson I when the binary link of the loops with four links is fixed and the adjacent links are ternary, Stephenson II, when the binary link of the five link loop is fixed and has an ternary adjacent link and Stephenson III when an ternary link is fixed. Although the synthesis of these types of mechanisms (reffering to the Watt and Stephenson mechanisms with constant lengths) on the basis of the prescribed positions was and still is discussed by specialists [1], [2], [4] the present paper presents as a innovation in the way of solving the synthesis problem for six mechanisms, which have an adjustable in steps length element, by developing the mathematical model on the basis of the input-output equation for the two adjustments.

2. THE ESTABLISHMENT OF THE INPUT – OUTPUT EQUATION

The input – output equation for the synthesis of the six-link mechanisms which generate prescribed position are determined on the basis of the method presented by Dhingra and Mani [1]. For the first loop the input – output equation may be written:

$$A\,sin\varphi + B\,cos\varphi + C = 0, \qquad (1)$$

where φ is the output angle of the loop, and A, B and C depend of the loop input angle. For the second loop the input – output equation may be written the same way:

$$A_1\,sin\varphi + B_1\,cos\varphi + C_1 = 0 \qquad (2)$$

where φ, in this case, is the loop input angle, and ***A, B, C*** depend on the loop output angle.

Eliminating φ from equations (1) and (2) and using the known trigonometrical relation:

$$sin^2\varphi + cos^2\varphi = 1 \qquad (3)$$

results the input – output equation for the synthesis of the six link mechanisms for prescribed positions:

$$\begin{aligned}&\left(A_iB_{1i} - A_{1i}B_i\right)^2 - \left(A_{1i}C_i - A_iC_{1i}\right)^2 - \\ &-\left(B_iC_{1i} - B_{1i}C_i\right)^2 = 0;\ i = \overline{1,n}\end{aligned} \quad ,(3)$$

where A, B, C, A_1, B_1 and C_1 depend on the imposed prescribed positions.

As it follows it is presented the shape of equation (3) for the Watt I and Stephenson I mechanisms which have an adjustable in steps length link.

3. ANALYTICAL MODEL FOR THE SYNTHESIS OF THE WATT I MECHANISM

It is considered the Watt I mechanism in figure 1. It's link 3 has the length adjustable in steps.

The dimensionless parameters which define the mechanism in figure 1 have the following rotations: $AD = s_1$, $DE = s_{12}$, $CD = s_2$, $BC = s_3$, $BC^* = s_3^*$, $DG = s_{21}$, $EG = s_4$, $AB = s_6 = 1$.

Characteristic for the link is the fact that it has two lengths depending on the working phase: a minimal one $s_3 = s_{3\,min}$ and a maximal one

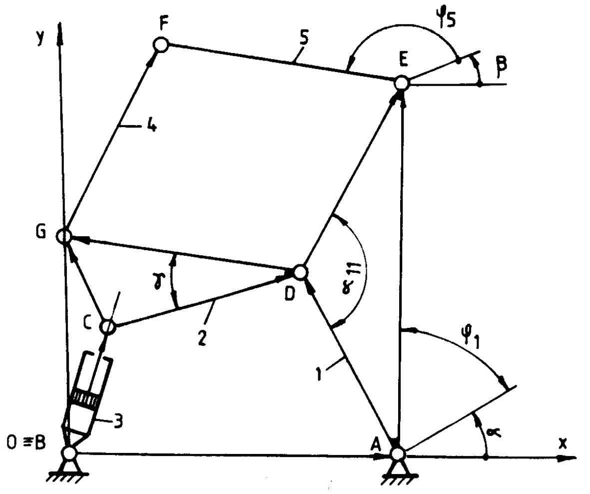

Figure1. Watt I mechanism with an adjustable in steps links

$s_3^* = s_{3\,max}$. In this case, the vector of the variable synthesis parameters has the following components:

$$\bar{u}=\bar{u}\left(s_1,s_{12},s_{21},\alpha_{11},s_2,s_3,s_3^*,s_4,s_5,\alpha,\beta,\gamma\right) \tag{4}$$

Input – output equation (3) for the Watt I mechanism with an adjustable links is:

$$\begin{aligned}&(A_iB_{1i}-A_{1i}B_i)^2-(A_{1i}C_i-A_iC_{1i})^2-\\&-(B_iC_{1i}-B_{1i}C_i)^2=0;\ i=\overline{1,k}\\&(A_jB_{1j}-A_{1j}B_j)^2-(A_{1j}C_j^*-A_jC_{1j})^2-\\&-(B_jC_{1j}-B_{1j}C_j)^2=0;\ j=\overline{k+1,n}\end{aligned} \tag{5}$$

For the determining of ***A, B, C, A_1, B_1, C_1 , C^**** are used the equations of the vectorial loops ***ABCDA***, ***CEFGC*** and ***ABC*DA***

$$\begin{aligned}&\bar{s}_6+\bar{s}_1=\bar{s}_3+\bar{s}_2\\&\bar{s}_6+\bar{s}_1=\bar{s}_3^*+\bar{s}_2\\&\bar{s}_{21}+\bar{s}_4=\bar{s}_{12}+\bar{s}_5\end{aligned} \tag{6}$$

Using the projections of the vectorial equations (6), on the axes of the coordinate system, results the relations which define ***A, B, C, A_1, B_1, C_1, C^****, relations presented in appendix 1.

4. ANALYTIC MODEL FOR THE SYNTHESIS OF THE STEPHENSON I MECHANISMS

The Stephenson I mechanism with the link 4 adjustable in presented in figure 2.The dimensionless synthesis parameters of the mechanism are: $AE = s_1$, $AB = s_{11}$, $BC = s_2$, $CD = s_{31}$, $DG = s_3$, $EF = s_4$, $EF = s_4$, $EF^* = s_4^*$, $FG = s_5$, $AD = s_6 = 1$. The link 4 has two relative dimensions depending on the working phase: $s_4 = s_{4\,min}$, $s_4 = s_{4\,max}$ (relative dimensions depending on the working of the hydraulic cylinder 4).

The vector of the variable synthesis parameters has the following components:

$$\bar{u}=\bar{u}\left(s_{11},s_1,\alpha_{11},s_2,s_3,s_{31},s_4,s_4^*,s_5,\alpha,\beta\right) \tag{7}$$

The input – output equation is determined on the basis of the vectorial equations written for the loops ***ABCDA, AEFGDA, AEF*GDA***

$$\begin{aligned}&\bar{s}_{11}+\bar{s}_2=\bar{s}_6+\bar{s}_{31}\ ;\\&\bar{s}_1+\bar{s}_5=\bar{s}_6+\bar{s}_3+\bar{s}_4\ ;.\\&\bar{s}_1+\bar{s}_5=\bar{s}_6+\bar{s}_3+\bar{s}_4^*\ ;\end{aligned} \tag{8}$$

and has the following shape:

$$\begin{aligned}&(A_iB_{1i}-A_{1i}B_i)^2-(A_{1i}C_i-A_iC_{1i})^2-\\&-(B_iC_{1i}-B_{1i}C_i)^2=0;\ i=\overline{1,k}\end{aligned}\ ;$$

$$\begin{aligned}&(A_jB_{1j}-A_{1j}B_j)^2-(A_{1j}C_j-A_jC_{1j}^*)^2-\\&-(B_jC_{1j}^*-B_{1j}C_j)^2=0;\ j=\overline{k+1,n}\end{aligned}\ . \tag{9}$$

The prescribed positions are given by the pairs of angles (φ_1,φ_5). The coefficinets of the equations system (9) are determined with the help of the coordinate axes projections of the vectorial equations (8). The calculation relations for these ones are presented in appendix 2.

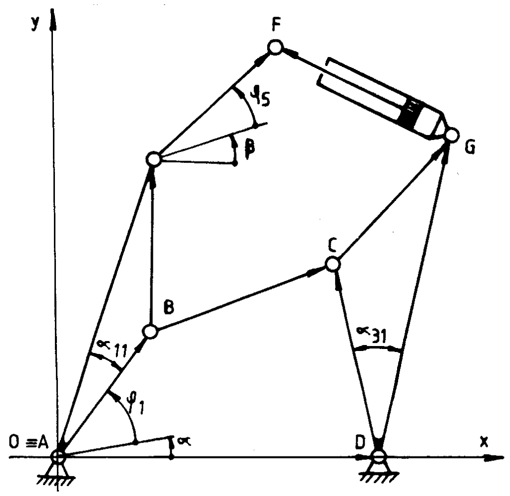

Figure 2. The Stephenson I mechanism with an adjustable in steps link

5. CONCLUSIONS

- The input – output equations for the mechanisms with adjustable in steps links is written for each working phase. For these phases the hydraulic cylinder, which is the adjustable link, has a minimal and a maximal dimension.
- The dimension of the vector of the variable synthesis parameters depends on the number of the adjustable in steps links.
- The shape of the mathematical model depends on the number of the prescribed possitions.

References

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APPENDIX 1

The calculation of the coefficinets from the input output equation for the Watt I mechanism with an adjustable link.

The calculation relations for ***A, B, C, A₁, B₁, C₁*** and C^* are:

$$A = -2s_1 s_2 \sin(\varphi_1 + \alpha) ;$$
$$B = -2s_2 - 2s_1 s_2 \cos(\varphi_1 + \alpha);$$
$$C = 1 + s_1^2 + s_2^2 - s_3^2 + 2s_1 \cos(\varphi_1 + \alpha);$$
$$A_1 = M_1 + M_2 + M_3 + M_4 ,$$

where

$$M_1 = -2s_{21}s_{12} \sin\gamma \cos(\varphi_1 + \alpha)\cos\alpha_{11} + \\ + 2s_{21}s_{12} \sin(\varphi_1 + \alpha)\sin\gamma \sin\alpha_{11} ;$$

$$M_2 = 2s_5 s_{21} \sin\gamma \cos\varphi_5 \cos\beta - \\ - 2s_5 s_{21} \sin\gamma \sin\varphi_5 \sin\beta ;$$

$$M_3 = -2s_{21}s_{12} \cos\gamma \sin(\varphi_1 + \alpha)\cos\alpha_{11} - \\ - 2s_{21}s_{12} \cos\gamma \cos\varphi_1 \sin\alpha_{11} ;$$

$$M_4 = 2s_{21}s_5 \cos\gamma \sin\varphi_5 \cos\beta + \\ + 2s_{21}s_5 \cos\gamma \sin\beta \cos\varphi_5 ;$$

$$B_1 = N_1 + N_2 + N_3 + N_4 ,$$

with

$$N_1 = -2s_{21}s_{12} \cos\gamma \cos(\varphi_1 + \alpha)\cos\alpha_{11} + \\ + 2s_{21}s_{12} \cos\gamma \sin(\varphi_1 + \alpha) ;$$

$$N_2 = 2s_5 s_{21} \cos\gamma \cos\varphi_5 \cos\beta - \\ - 2s_5 s_{21} \cos\gamma \sin\varphi_5 \sin\beta ;$$

$$N_3 = 2s_{21}s_{12}\sin\gamma\sin(\varphi_1+\alpha)\cos\alpha_{11} + \\ + 2s_{21}s_{12}\sin\gamma\cos(\varphi_1+\alpha)\sin\alpha_{11} \;;$$

$$N_4 = -2s_{21}s_5\sin\gamma\sin\varphi_5\cos\beta - \\ - 2s_{21}s_5\sin\gamma\sin\beta\cos\varphi_5 \;;$$

$$C_1 = P_1 + P_2 + P_3 \;,$$

from which

$$P_1 = s_{21}^2 + s_{12}^2 + s_5^2 - s_4^2 \;;$$

$$P_2 = -2s_{12}s_5\cos(\varphi_1+\alpha+\alpha_{11})\cos(\varphi_5+\beta) \;;$$

$$P_3 = -2s_{12}s_5\sin(\varphi_1+\alpha+\alpha_{11})\sin(\varphi_5+\beta) \;;$$

$$C^* = 1 + s_1^2 + s_2^2 - s_3^{*^2} + 2s_1\cos(\varphi_1+\alpha).$$

The dimensionless parameters s_{22} = CG and s_{13} = AE are calculated with the relations:

$$s_{22} = \sqrt{s_2^2 + s_{21}^2 - 2s_2s_{21}\cos\gamma} \;;$$
$$s_{13} = \sqrt{s_1^2 + s_{12}^2 - 2s_1s_{12}\cos\alpha_{11}} \;.$$

APPENDIX 2

The calculation of the coefficients from the input – output equation for the Stephenson I mechanism adjustable linke.

The calculation relations for ***A, B, C, A₁, B₁, C₁*** and C_1^* have the following shape:

$$A = D_1 + D_2 \,,$$

where

$$D_1 = 2s_{11}s_{31}\cos(\varphi_1+\alpha)\sin\alpha_{31} - 2s_{31}\sin\alpha_{31} \;;$$

$$D_2 = -2s_{11}s_{31}\sin(\varphi_1+\alpha)\cos_{31} \;;$$

$$B = E_1 + E_2 \;,$$

with

$$E_1 = -2s_{11}s_{31}\cos(\varphi_1+\alpha)\sin\alpha_{31} + 2s_{31}\cos\alpha_{31} \;;$$

$$E_2 = -2s_{11}s_{31}\sin(\varphi_1+\alpha)\sin\alpha_{31} \;;$$

$$C = s_{11}^2 + s_{31}^2 + 1 - s_2^2 - 2s_{11}\cos(\varphi_1+\alpha) \;;$$

$$A_1 = -2s_1s_3\sin(\varphi_1+\alpha+\alpha_{11}) - \\ - 2s_3s_5\sin(\varphi_5+\beta) + 2s_3 \;;$$

$$B_1 = -2s_1s_3\cos(\varphi_1+\alpha+\alpha_{11}) - \\ - 2s_3s_5\cos(\varphi_5+\beta) + 2s_3 \;;$$

$$C_1 = F_1 + F_2 + F_3 \;,$$

from which

$$F_1 = s_1^2 + s_5^2 + 1 - s_3^2 - s_4^2 + \\ + 2s_1s_5\cos(\varphi_1+\alpha+\alpha_{11})\cos(\varphi_5+\beta) \;;$$

$$F_2 = -2s_1\cos(\varphi_1+\alpha+\alpha_{11}) - \\ - 2s_5\cos(\varphi_5+\beta) \;;$$

$$F_3 = 2s_1s_5\sin(\varphi_1+\alpha+\alpha_{11})\sin(\varphi_5+\beta) \;;$$

$$C_1^* = F_1^* + F_2 + F_3 \;,$$

and

$$F_1^* = s_1^2 + s_5^2 + 1 - s_3^2 - s_4^{*2} + \\ + 2s_1s_5\cos(\varphi_1+\alpha+\alpha_{11})\cos(\varphi_5+\beta) \,.$$

The parameters s_{12} and s_{13} are calculated with the relations:

$$s_{12} = \sqrt{s_1^2 + s_{11}^2 - 2s_1s_{11}\cos\alpha_{11}} \;;$$

$$s_{13} = \sqrt{s_3^2 + s_{31}^2 - 2s_3s_{11}\cos\alpha_{31}} \;.$$